1.0 | 4.5 5.0 | 2.8 | 2.5
1.1 | | 3.2 | 2.2
| | 3.6 | 2.0
| | 4.0 | 1.8
1.25 | 1.4 | 1.6

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ACTA AERONAUTICA ET ASTRONAUTICA SINICA

(Selected Articles)

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APPLICATION OF HARMONIC ANALYSIS METHOD
TO RESEARCH ON ROTOR AIRLOADS

Qiu Zhenhan

(*Chinese Helicopter Research and Development Institute*)

Abstract

According to the rotor vortex theory, the rotor circulation and the rotor induced velocity are developed into Fourier series. The circulation distribution along blade spanwise is expressed in Terms of segment-by-segment linear functions. In consequence the induced velocity equations and the circulation equations are derived. The engineering application of the rotor vortex theory is provided. Then the induced velocity and its harmonic components are obtained to provide a quantitative basis for the vortex model. For calculating each order harmonic components of the induced velocity a simplified method is put forward which considers the effects of each order circulation with neglecting those of higher order. The method saves the computer time and is of significant benefit.

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APPLICATION OF HARMONIC ANALYSIS METHOD TO RESEARCH ON ROTOR
AIRLOADS

Qin Zhenhan
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This paper uses the rotor vortex theory
of Professor Wang Shichuen as a basis to develop
the rotor circulation and induced velocity into
Fourier series. The circulation distribution
along blade spanwise is expressed in terms of
segment-by-segment linear functions and the
linear equations for calculating rotor airloads
using the circulation as the unknown are derived.
This has resolved the practical application prob-
lems of the rotor vortex theory that for many
years have not been able to be resolved.

I. Preface

There have been many theories and articles on the aerodynamic

calculations of helicopter rotor-blades, and among which is the

rotor vortex theory of the stationary vortex system of Professor

Wang Shicun[1]. But it requires proper modifications when applying

this theory to engineering calculations. Due to the limitation

of resources in the past, the rotor-blade could only be treated

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as a rigid blade, and this is obviously improper. As the application
of computer develops in our country, the elastic deformation (bending,
twisting, etc.) of the rotor-blade can be considered simultaneously
and this can more truthfully reflect the rotor characteristics.

According to Reference [1], this paper develops the rotor circula-
tion and induced velocity into the Fourier series. This manipulation
makes the physical significance between the circulation and its
related parameters become very clear and is rather conducive toward
the understanding of the vortex model and the effects of various
vortex systems. Although the mathematical manipulations are quite
complex, a smaller computer capacity is required. While high accuracy
computations can be conducted on a computer with large capacity,
results can also be obtained using a computer with small capacity.
For example, it only requires half an hour to obtain results that
meet engineering requirements on the DJS-21 computer which only
processes ten-thousand operations per second. Therefore, it is
economical and suitable for engineering applications.

II. Induced Velocity Equations

According to Reference [1], the axial induced velocity at any
point on the rotor hub can be developed into the Fourier series

$$v_p(\bar{r}) = v_{0p}(\bar{r}) + \sum_{n=1}^{N} [v_{anp}(\bar{r})\cos n\psi + v_{anp}(\bar{r})\sin n\psi] \qquad (1)$$

where $v_{0p}(\bar{r}) = v_{P_0} + v_{Z_0} + v_{H_0};$ $v_{anp}(\bar{r}) = v_{Pan} + v_{Zan} + v_{Han};$ $v_{anp}(\bar{r}) = v_{Pan} + v_{Zan} + v_{Han}$
Here every induced velocity is agitated by the circulation which
consists of attached vortex, vertical free vortex and horizontal
free vortex.

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According to Reference [2], if the circulation distribution along spanwise is expressed in terms of segment-by-segment linear functions, the induced velocity derived from vertical free vortex can be expressed as

$$v_z = v_{z_0} + \sum_{n=1}^{N} (v_{z_{nc}}\cos n\psi + v_{z_{ns}}\sin n\psi) \tag{2}$$

where

$$v_{z_0} = -K_v\Gamma_0(\bar{r}) + \sum_{m=1}^{N}\left\{\sum_{t=1}^{P-1}\left[(C_{13}[m]C_{f13}/\bar{r})\Delta\Gamma_{mc} + (C_{16}[m]C_{f16}\right.\right.$$

$$\left.- C_{16}[m]C_{f16})\Delta\Gamma_{mc}\right]/\delta_t + \sum_{t=p}^{NN}\left[(C_{16}[m]C_{f16}/\bar{r})\Delta\Gamma_{mc}\right.$$

$$\left.\left.+ C_{17}[m](C_{f17}-C_{f16})\Delta\Gamma_{mc}\right]/\delta_t\right\}$$

$$v_{z_{nc}} = \sum_{t=1}^{P-1}[E_1 C_{f6}\Delta\Gamma_6 - (1/2)K_v C_{f6}\Delta\Gamma_{nc} + (E_4 C_{f1}/\bar{r})\Delta\Gamma_{nc}]/\delta_t$$

$$+ \sum_{t=p}^{NN}[(1/2)K_v C_{f7}\Delta\Gamma_{nc} - E_2 C_{f11}\Delta\Gamma_6 + (E_4 C_{f3}/\bar{r})\Delta\Gamma_{nc}]/\delta_t$$

$$+ \sum_{m=1}^{N}\left\{\sum_{t=1}^{P-1}[(C_1[m]C_{f16} + C_3[m]C_{f26})\Delta\Gamma_{mc} - (C_9[m]C_{f13}/\bar{r})\Delta\Gamma_{mc}]/\delta_t\right.$$

$$\left.+ \sum_{t=p}^{NN}[(C_5[m]C_{f21} + C_7[m]C_{f22})\Delta\Gamma_{mc} - (C_{11}[m]C_{f24}/\bar{r})\Delta\Gamma_{mc}]/\delta_t\right\}$$

$$v_{z_{ns}} = \sum_{t=1}^{P-1}[(E_3 C_{f16}/\bar{r})\Delta\Gamma_6 - (1/2)K_v C_{f6}\Delta\Gamma_{ns} + (E_4 C_{f2}/\bar{r})\Delta\Gamma_{ns}]/\delta_t$$

$$+ \sum_{t=p}^{NN}[(E_6 C_{f12}/\bar{r})\Delta\Gamma_6 + (1/2)K_v C_{f7}\Delta\Gamma_{ns} + (E_4 C_{f3}/\bar{r})\Delta\Gamma_{ns}]/\delta_t$$

$$+ \sum_{m=1}^{N}\left\{\sum_{t=1}^{P-1}[(C_2[m]C_{f16} + C_4[m]C_{f26})\Delta\Gamma_{ns} - (C_{10}[m]C_{f23}/\bar{r})\Delta\Gamma_{ns}]/\delta_t\right.$$

$$\left.+ \sum_{t=p}^{NN}[(C_6[m]C_{f21} + C_8[m]C_{f22})\Delta\Gamma_{ns} - (C_{12}[m]C_{f24}/\bar{r})\Delta\Gamma_{ns}]/\delta_t\right\}$$

where $\quad K_v = k/(4\pi V_1); \quad \Delta\Gamma = \Gamma_{t+1} - \Gamma_t; \quad \delta_t = \bar{r}_{t+1} - \bar{r}_t$

E_i and $C_j[m]$ are the quantites related to K_v and the harmonic order number m and n. For example

$$E_1 = K_v n C^n$$

$$C_1[m] = \{C^n[(2-m-n)/2]_{m-1} + \delta C^{-n}[(2+n-m)/2]_{m-1}\}$$

$$\times C^n K_v/[2(m-1)!]$$

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where $C = (1 - \sin|\alpha_1|)/\cos|\alpha_1|$

C_{jk} is the quantity related to the integration of the super geometrical function [3]

$$C_l(\iota,\ l,\ \alpha,\ \beta,\ \bar{r},\ 0) = \int_{\rho_l}^{\bar{r}_{l+1}} (\bar{\rho}/\bar{r})^l F[\alpha,\ \beta,\ \gamma,\ (\bar{\rho}/\bar{r})^2] d\bar{\rho}$$

For example

$$C_{l13} = C_l(\iota,\ m,\ (m+1)/2,\ (m+1)/2,\ \bar{r},\ 0)$$

The expressions of induced velocities v_F and v_H agitated by the attached vortex and horizontal free vortex are similar to Equation (2).

III. Circulation Equations

According to Reference [2], the circulation is developed into Fourier series

$$\Gamma = \Gamma_0 + \sum_{m=1}^{N} (\Gamma_{mc}\cos m\psi + \Gamma_{ms}\sin m\psi) \tag{3}$$

where

$$\Gamma_0 = (a_\infty \bar{b}/2)\left\{v_0 + \bar{r}(\phi_0 + \phi_\Delta) + (\mu/2)\phi_2 + \lambda_0\right.$$

$$\left. + \sum_{j=0}^{j1} \{K[(\mu/2)b_1^j - \bar{r}a_0^j]dy^j(\epsilon)/d\bar{r} + (\mu/2)a_1^j dy^j(\bar{r})/d\bar{r}\}\right\}$$

$$\Gamma_{mc} = (a_\infty \bar{b}/2)\left\{v_{mc} + \sum_{j=0}^{j1}\left\{K[\bar{r}a_m^j + (\mu/2))(b_{m-1}^j - b_{m-1}^j)]dy^j(\bar{e})/d\bar{r}\right.\right.$$

$$\left.\left. + (\mu/2)(a_{m-1}^j + a_{m+1}^j)dy^j(\bar{r})/d\bar{r} + mb_m^j y^j(\bar{r})\right\}\right\}$$

$$\Gamma_{ms} = (a_\infty \bar{b}/2)\left\{v_{ms} + \sum_{j=0}^{j1} \{K[\bar{r}b_m^j + (\mu/2)(a_{m-1}^j - a_{m+1}^j)]dy^j(\bar{e})/d\bar{r}\right.$$

$$\left. + (\mu/2)(b_{m-1}^j + b_{m+1}^j)dy^j(\bar{r})/d\bar{r} - ma_m^j y^j(\bar{r})\}\right\}$$

The circulation distribution along spanwise is expressed in terms of segment-by-segment linear functions, then the circulation equation in the form of Γ_{op} is

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$$\Gamma_{oo}/A_o + v_{oo} + \sum_{t=1}^{NN} \sum_{j=0}^{j1} \left\{ \mu(\bar{r}B_{sr} - \mu K/2)dy^j(\bar{e})/d\bar{r}\,a_s^t T_{1t}^i \Gamma_{ot} \right.$$

$$+ \{(\mu/2)\bar{r}B_{sj}^t a_s^t T_{1t}^i - (\bar{r}B_{sj}^t + (\mu/2)dy^j(\bar{r})/d\bar{r})a_s^t T_{st}^i\}\Gamma_{1ct}$$

$$+ [(\mu/2)\bar{r}B_{ss}a_s^t T_{1t}^i - (\mu K/2 - \bar{r}B_{ss})a_s^t T_{1t}^i]dy^j(\bar{e})/d\bar{r}\,\Gamma_{1st}$$

$$+ [(\mu/2)(\mu K/2 - \bar{r}B_{ss})a_s^t T_{1t}^i - \bar{r}B_{ss}a_s^t T_{1t}^i]dy^j(\bar{e})/d\bar{r}\,\Gamma_{2ct}$$

$$+ \{\bar{r}B_{sj}^t a_s^t T_{st}^i - (\mu/2)(\bar{r}B_{sj}^t + (\mu/2)dy^j(\bar{r})/d\bar{r}\,a_s^t T_{1t}^i)\}\Gamma_{2st}$$

$$\left. - (\mu/2)\bar{r}B_{sj}^t a_s^t T_{1t}^i \Gamma_{sct} - (\mu/2)\bar{r}B_{ss}dy^j(\bar{e})/d\bar{r}\,a_s^t T_{1t}^i \Gamma_{sst} \right\}$$

$$= \bar{r}B_{oj} + \Delta\Psi\bar{r}^2 + \mu\theta_s/2 + \lambda_o + \bar{r}B_{sj}C_\tau/(8XK_\rho) \tag{4}$$

where $\quad A_o = a_\infty\bar{b}/2, \quad K_o = k/(4\pi),$

$$B_{oi}^j = B_{1r}\int_{\rho_1}^1 y^j d\bar{r} - B_{2s}(1 - \bar{r}_1 y^j(\bar{r})); \quad B_{sj}^t = B_{sr}(1 - y^j(\bar{r}_1)),$$

$$T_{st}^i = \begin{cases} (\bar{r}_2 Y_{s,1}^i - Y_{(s+1),1}^i)/\delta_1, & t = 1 \\ (\bar{r}_{t+1}Y_{s,t}^i - Y_{(s+1),t}^i)/\delta_t + (Y_{(s+1),(t-1)}^i - \bar{r}_{(t-1)}Y_{s,(t-1)}^i)/\delta_{t-1} & 2 \leq t \leq N_1 \\ (Y_{(s+1),N_1}^i - \bar{r}_{N_1}Y_{s,N_1}^i)/\delta_{N_1} & t = N_1 + 1 \end{cases}$$

$$Y_{s,t}^i = \int_{\rho_t}^{\rho_{t+1}} y^j \bar{\rho}^{s-1} d\bar{\rho}$$

B_{ns} is the quantity related to B_o, B_s and motion parameters. For example $\quad B_{1r} = \mu(B_o + B_r)/\{2[(B_o/3)(1 - \bar{r}_1^3) + (\mu^2/2)(1 - \bar{r}_1)B_r]\}$

where $\quad B_o = 1/(1 + A_b K_v), \quad B_r = 1/[1 + A_b K_v(1 + C^2)]$

Γ_{1op}, Γ_{1sp} and other circulation equations can be derived accordingly.

IV. Rotor-Blade Airloads

Each order harmonic component of circulation can be obtained from the linear algebraic equations with circulation as the unknown which are derived by substituting the induced velocity equations into the circulation equations. According to the Zhukovskiy formula, the airloads per unit rotor-blade length are:

$$dT_1/d\bar{r} = (\rho R^2/m_1)(W_o\Gamma) \tag{5}$$

where

$$W_x = \bar{r} + \mu\sin\psi, \quad m_1 = \text{mass of one blade;} \quad \Gamma = \Gamma_0 + \sum_{m=1}^{N}(\Gamma_{mc}\cos m\psi + \Gamma_{ms}\sin m\psi)$$

Thus, the expressions of each order airloads per unit rotor-blade length are obtained; and therefore the airloads are obtained.

$$(dT_1/d\bar{r})_0 = (\rho R^3/m_1)(\bar{r}\Gamma_0 + (\mu/2)\Gamma_{1s})$$

$$(dT_1/d\bar{r})_{1c} = (\rho R^3/m_1)(\bar{r}\Gamma_{1c} + (\mu/2)\Gamma_{2s})$$

$$(dT_1/d\bar{r})_{1s} = (\rho R^3/m_1)(\mu\Gamma_0 + \bar{r}\Gamma_{1s} - (\mu/2)\Gamma_{2c})$$

$$(dT_1/d\bar{r})_{mc} = (\rho R^3/m_1)(\bar{r}\Gamma_{mc} + (\mu/2)(\Gamma_{(m+1)s} - \Gamma_{(m-1)s}))$$

$$(dT_1/d\bar{r})_{ms} = (\rho R^3/m_1)(\bar{r}\Gamma_{ms} + (\mu/2)(\Gamma_{(m-1)c} - \Gamma_{(m+1)c}))$$

V. Examples and Discussions

Numerical computations were conducted using the H-34 helicopter as an example. The calculated values were compared with the measured values during actual flight and the results are satisfactory; see Figs. 1 and 2.

Another helicopter with takeoff weight of 14,400 kg and rotor diameter f 21.3 m was used as an example to calculate each component of the induced velocity and obtain the curves of the azimuthal variation of total axial induced velocity (Fig. 3). Due to the limitation in space, only the second order components of the induced velocities which are agitated by separate harmonic components of attached vortex, vertical free vortex and horizontal free vortex, are given. See Table 1-3. The obtained harmonic components of the induced velocities have not yet been seen in any references. They provide a quantitative basis for the analysis and research of the vortex model.

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It is very beneficial to perform example computations when only considering the same order induced velocity which is agitated by each order harmonic circulation before the same order, as shown in Fig. 4. If only the shaded areas are considered, comparison between the obtained results and the effects to the induced velocity when the total circulation is considered shows that their numerical values are very close; see Fig. 5. The computation time, however, is greatly reduced. The higher the order considered, the more distinctive the effect of reduction of computation time.
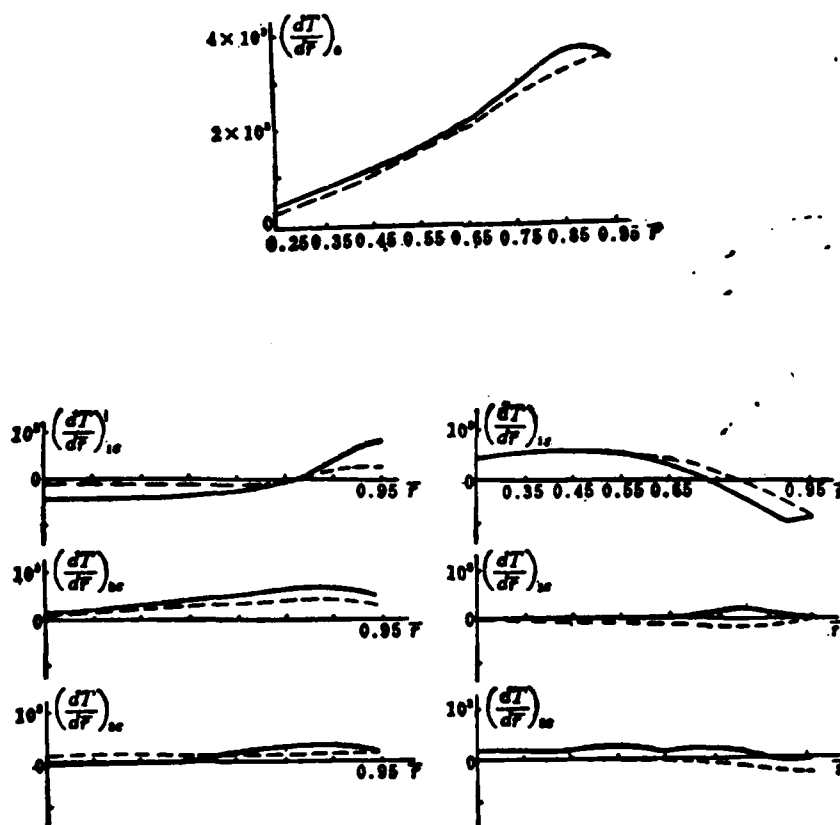

Fig. 5. Comparison of the harmonic components of $dT/d\bar{r}$(N/m) at $\mu = 0.2$
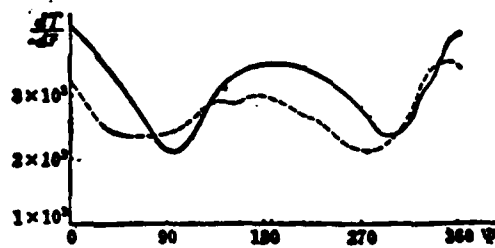——test; -----Calculation.

Fig. 2. Comparison of the azimuthal variation of $dT/d\Psi$ (N/m) at $\mu = 0.2$, for $\bar{r} = 0.75$
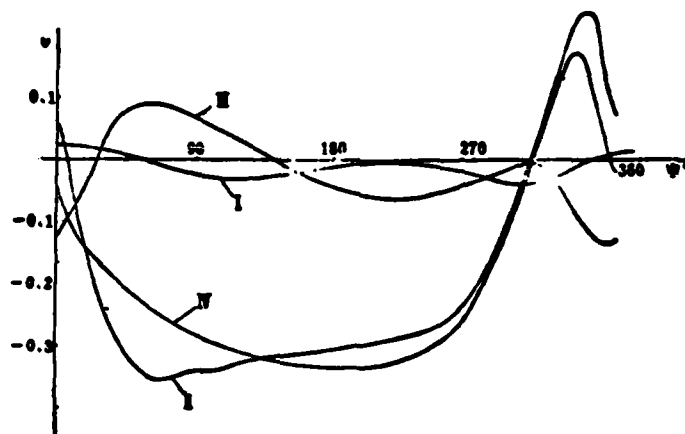——test; ----- calculation.

Fig. 3. Azimuthal variation of the induced velocity v at $\mu = 0.30$, for $\bar{r} = 0.75$
I —The induced velocity v_P by bound vortices; II —The induced velocity v_Z by longitudinal vortices;
III —The induced velocity v_H by lateral vortices; IV —Total induced velocity.

Table 1   The harmonic components of the induced velocity v_P

$\bar{r}$	v_{P0}	$v_{P1c} \times 10^{-2}$	$v_{P1s} \times 10^{-2}$	$v_{P2c} \times 10^{-2}$	$v_{P2s} \times 10^{-2}$
0.25	0	−0.998	−0.250	0.462	0.694
0.35	0	−0.283	0.0318	0.937	0.995
0.45	0	0.423	0.126	0.855	0.749
0.55	0	0.553	0.113	0.764	0.629
0.65	0	0.601	0.0925	0.665	0.518
0.75	0	0.588	0.0719	0.558	0.414
0.85	0	0.513	0.0538	0.433	0.311
0.95	0	0.418	0.0448	0.338	0.245

Table 2   The harmonic components of the induced velocity v_z

$\bar{r}$	$v_{z0} \times 10^{-1}$	$v_{z1c} \times 10^{-1}$	$v_{z1s} \times 10^{-2}$	$v_{z2c} \times 10^{-2}$	$v_{z2s} \times 10^{-2}$
0.25	−0.0827	0.104	−0.502	0.0535	−0.0578
0.35	−0.124	0.153	−0.933	0.396	−0.360
0.45	−0.186	0.167	−0.922	0.680	−0.533
0.55	−0.212	0.135	−0.811	0.760	−0.564
0.65	−0.233	0.138	−0.660	0.808	−0.569
0.75	−0.250	0.118	−0.474	0.830	−0.545
0.85	−0.263	0.0942	−0.256	0.833	−0.479
0.95	−0.268	0.0739	−0.146	0.832	−0.399

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Table 3 The harmonic components of the induced velocity v_H

ϕ	$v_{H0} \times 10^{-2}$	$v_{H1c} \times 10^{-2}$	$v_{H1s} \times 10^{-2}$	$v_{H2c} \times 10^{-2}$	$v_{H2s} \times 10^{-2}$
0.25	−0.242	0.538	−0.572	−0.171	0.00510
0.35	−0.132	0.471	−0.280	−0.201	−0.000109
0.45	−0.0530	0.342	0.114	−0.240	0.0695
0.55	−0.0282	0.285	0.290	−0.257	0.122
0.65	−0.00967	0.231	0.450	−0.272	0.161
0.75	0.00296	0.179	0.592	−0.285	0.194
0.85	0.00959	0.130	0.715	−0.295	0.221
0.95	0.0107	0.109	0.768	−0.298	0.230

Fig.4 Relation between the induced velocity and the Circulation in simplified calculation
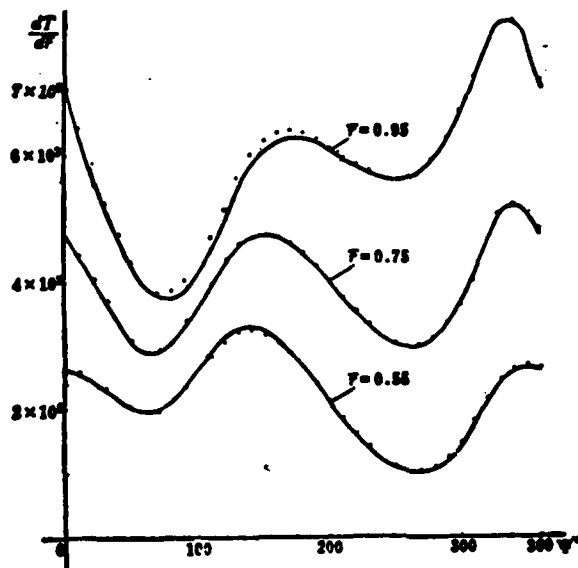

Fig.5 Variation of $dT/d\phi$ (N/m) with azimuth at $\mu = 0.27$

——simplified circulation, ······total circulation.

(Xuie Zhenzhong and Duan Yuzhang took part in the work of this research subject).

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SOME PROBLEMS IN THE FINITE-DIFFERENCE COMPUTATION OF THREE-DIMENSIONAL TRANSONIC FLOWS

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(Beijing Institute of Aerodynamics)

Abstract

The mixed finite-difference relaxation iteration method is applied to calculation of the wing-body combination with rectangular wings based on the three-dimensional transonic small-disturbance potential equation in the cylinder coordinates. Meanwhile, the influences of different computation regions and relaxation parameters (on subsonic points) on the ranges of the computed Mach numbers, angles of attack and calculated results are studied.

It is shown that extending the computation region brings about increasing the convergence range of the small-disturbance equation. and the convergence rate and computation accuracy would be are enhanced if the subsonic relaxation parameter is taken to be 1.9.

Also, it is demonstrated that the calculated results differ when the finite or infinite region is taken as the computation region of the r-direction. Particularly the distribution of the pressure cooeficients near the wingtip varies obviously as the incident Mach number becomes larger.

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SOME PROBLEMS IN THE FINITE-DIFFERENCE COMPUTATION OF THREE-
DIMENSIONAL TRANSONIC FLOWS

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I. Introduction of Basic Method

1. Three-Dimensional Transonic Small-Disturbance Equation

The three-dimensional transonic small-disturbance equation in
cylindrical coordinates is.

$$A\varphi_{zz} + \varphi_{rr} + \frac{1}{r}\varphi_r + \frac{1}{r^2}\cdot\varphi_{\theta\theta} = 0 \qquad (1)$$

where φ is disturbance potential; $A = \beta^2 - (\gamma + 1)M_\infty^2\varphi_z$, $\beta^2 = 1 - M_\infty^2$, γ
is specific heat; M_∞^2 is the incident Mach number. If r-direction
is selected as infinite region, through coordinate conversion
$\eta = br/(1 + ar)$, the infinite region of r is converted to the finite
computation region η . Here a and b are constants to be determined.
Now Equation (1) can be rewritten as

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$$A\varphi_{xx} + \frac{('b - a\eta)^2}{b}\left[\left(\frac{1}{\eta} - \frac{2a}{b}\right)\varphi_\eta \right.$$
$$\left. + \frac{b - a\eta}{b}\varphi_{\eta\eta}\right] + \frac{(b - a\eta)^2}{\eta^2}\varphi_{\theta\theta} = 0 \tag{2}$$

Then Equations (1) and (2) correspond to the finite and infinite regions (select proper a and b values) of r respectively.
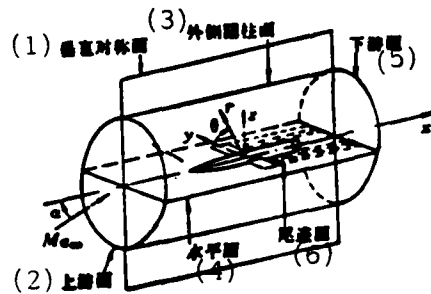

2. Boundary Conditions (See Fig. 1)

Fig. 1. Boundary surfaces and diagrammatic drawing of model
Key: (1)Vertical plane of symmetry; (2) Upstream plane; (3) Outer cylindrical surface; (4) Horizontal plane; (5) Downstream plane; (6) Tail-trace plane.

On the surface of the body: Assume the surface equation of the body symmetrical to the axes is r=R(x), then the boundary condition of the tangential flow is

$$\lim_{r \to R}(r\varphi_r) = R(x)(R'(x) - \alpha\sin\theta) \tag{3}$$

where α is the incident attack angle.

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On the surface of the wing: Assume the equations of upper and lower wing surfaces are $Z = f_\pm(x, y)$, then the flow along the upper and lower wing surfaces should satisfy

$$\lim_{\theta \to 0_\pm^z} (\varphi_\theta) = r \, [f'_\pm(x, y) - \alpha] \tag{4}$$

Here $f'_\pm(x, y) = \partial f_\pm(x, y)/\partial x$; $0_\pm^z$ represents the upper and lower wing surfaces.

On the tail-trace surface: It is known from the Kutta condition that

$$\varphi(x, y, 0_+) - \varphi(x, y, 0_-) = \Gamma(y) \tag{5}$$

There are two conditions on the upstream plane and the outer cylindrical surface:

(1) If Equation (1) is used, it should satisfy, from the solutions of small-disturbance wing lift problems,

$$\varphi(\Gamma) = \frac{\sin\theta}{4\pi r}\left(1 + \frac{x}{\sigma}\right)\int_{-s}^{s} \Gamma(y)dy \tag{6}$$

where $\sigma = \sqrt{x^2 + \beta^2 r^2}$, s is half of the span length.

(2) If Equation (2) is used, the disturbance potential is 0, i.e.,

$$\varphi = 0 \tag{7}$$

There are also two conditions on the downstream plane:

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(1) If Equation (1) is used,   should satisfy

$$\varphi(\Gamma) = \frac{r \sin\theta}{4\pi}\left(1 + \frac{x}{\sigma}\right)\int_{-s}^{s}\frac{\Gamma(y)}{\rho}dy \qquad (8)$$

where

$$\rho = \sqrt{r^2 - 2ry\cos\theta + y^2}.$$

(2) If Equation (2) is used, the disturbance velocity

$$\varphi_s = 0 \qquad (9)$$

3. Finite-Difference Formula, Finite-Difference Equation and Their Solutions

This paper uses the variable mixed finite-difference formula of Murman-Cole, i.e., φ_{xx} at subsonic point uses central finite-difference; at supersonic point uses incident finite-difference; r (or η) and θ-direction both use central finite-difference. The finite-difference formula at special point (with boundary conditions substituted):

(1) According to boundary condition (3), it can be obtained on the surface of the body that:

$$\frac{1}{r}(r\varphi_r)_r\Big|_{i,1,k} = \frac{1}{r_1\Delta r_1}\left[\frac{2r_1 + \Delta r_1}{2}\cdot\frac{\varphi_{i,2,k} - \varphi_{i,1,k}}{\Delta r_1} - R(R' - \alpha\sin\theta)\right] \qquad (10)$$

(2) On the upper and lower wing surfaces

$$\begin{cases} (\varphi_{\theta\theta})_{i,j,\theta_+} = \dfrac{2}{\Delta\theta_0}\left[\dfrac{\varphi_{i,j,1} - \varphi_{i,j,\theta_+}}{\Delta\theta_0} - r_j(f'_u - \alpha)\right] \\[4mm] (\varphi_{\theta\theta})_{i,j,\theta_-} = \dfrac{2}{\Delta\theta_{-1}}\left[r_j(f'_u - \alpha) - \dfrac{\varphi_{i,j,\theta_-} - \varphi_{i,j,-1}}{\Delta\theta_{-1}}\right] \end{cases} \qquad (11)$$

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(3) On the tail-trace plane, the pressure continuum and Kutta Condition are applied

$$(\varphi_{\theta\theta})_{i,j,\theta_-} = \frac{1}{\Delta\theta_{-1}} \left[\frac{(\varphi_{i,j,1} - \Gamma_j) - \varphi_{i,j,\theta_-}}{\Delta\theta_{-1}} - \frac{\varphi_{i,j,\theta_-} - \varphi_{i,j,\theta_-1}}{\Delta\theta_{-1}} \right] \quad (12)$$

Using the above finite-difference formulae, and on the plane of i=constant and the straight live of k=constant, the algebraic simultaneous equations with $\varphi_{i,j-1,k}$, $\varphi_{i,j,k}$ and $\varphi_{i,j+1,k}$ as unknowns are established. These equations are three-diagonal and can be solved by the chase method. Assume the solution is $\hat{\varphi}_i$, then the solution after relaxation is

$$\varphi_i^{n+1} = \omega\hat{\varphi}_i^{n+1} + (1 - \omega)\varphi_i^n \quad (13)$$

where ω is the relaxation parameter. The subsonic relaxation parameter ω_i in the computation is selected as 1.9 or 0.9; the supersonic relaxation parameter ω_2 is selected as 0.9. The iteration method is conducted to solve for the entire flow field. The iteration stops when the entire flow field satisfies

$$|\varphi_{i,j,k}^{n+1} - \varphi_{i,j,k}^n| < \varepsilon \quad (14)$$

where ε is the convergence range and is generally selected as $\varepsilon = 0.1 \times 10^{-4}$.

The computation formula for pressure coefficient on wing is

$$C_p = -2\varphi_x \quad (15)$$

On the surface of the body, the second-order term of the horizontal disturbance velocity is reserved

$$C_p = -2\varphi_x - \left(\varphi_r^2 + \frac{1}{r^2}\varphi_\theta^2\right) \quad (16)$$

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II. Example

1. Wing-Body Combination Equation

(1) The wing is a rectangular wing with double-arc airfoil

$$Z = \frac{2\delta}{b}\left(\frac{b^2}{4} - x\right) \tag{17}$$

The airfoil relative thickness ratio $\delta = 0.06$; chord length b=1 is a reference length; the aspect ratio $\lambda = L/b = 5.4$; L is the span length including the body.

2. The Body is the Model "AGARD-B" Body, the Equation for the Contour of the Head is

$$R(x) = \frac{1}{3}(5D+x)\left[1 - \frac{1}{9}(5D+x)^2 + \frac{1}{54}(5D+x)^3\right] \tag{18}$$
$$(-5D \leqslant x \leqslant 2D)$$

where D is the diameter of the cylindrical portion of the body. Assume D=1, when $x \geqslant 2$, then R=0.5 and extends into the downstream.

The origin of coordinates is selected at the intersection of the line connecting the center points of the chord and the body axes. The finite region of r is marked as I and the infinite region as II. The sampling points in the x, r (or η) and θ-direction are all distributed at nonequal distance.

III. computation Results

1. The maximum Mach number and attack angle in the computation region I are $M_{a\infty} = 0.926$ and $\alpha = 2^{O}$ respectively; in the computation region II they are $M_{a\infty} = 0.98$ and $\alpha = 4^{O}$ respectively. This indicates

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that the coordinate conversion $\eta = br/(1+ar)$ in the r-direction can increase the range of the calculated Mach number and attack angle of the small-disturbance equation.

2. Figure 2 gives the comparison curves between the computed results and the two-variable test results at $M_{a\infty} = 0.965$, $\alpha = 2^{\circ}$ and $\tilde{y} = 0.5$ ($\tilde{y}$ is the spanwise relative location on the wing cross-section). This figure shows that the conmputed results in region II are reasonable.
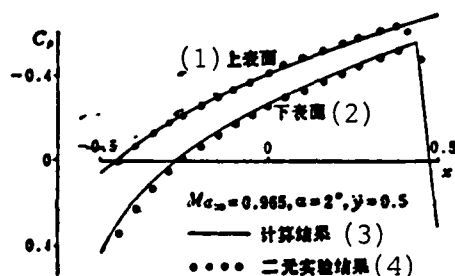

Fig. 2.   Comparison between computed results and test results
Key: (1) upper surface; (2) lower surface; (3) computed results; (4) two-dimensional test results.

3. Figure 3 shows that there is difference between the computed results of region I and II. At $\varepsilon = 0.1 \times 10^{-4}$, when the $M_{a\infty}$ is larger, there is a difference in the chordwise distribution of wing surface pressure coefficients and it becomes more obvious near the wingtip. This is because the outer boundary of region I is like a wall, causing the local Mach number to increase and the absolute value of C_p on the upper surface to be larger and that for the lower surface to be smaller.
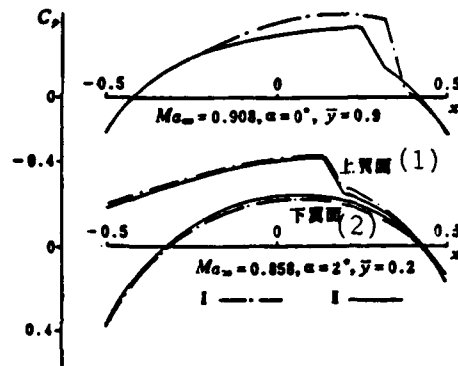
Fig. 3. Influence of computation regions on computed results
Key: (1) upper wing surface; (2) lower wing surface.

4. Figure 4 gives the curves of influence of subsonic relaxation parameters on computed results. Under the same ε, the computed results at $\omega_1 = 1.9$ is more accurate than that at $\omega_1 = 0.9$; its convergence rate is faster. There are similar phenomena in both computation region I and II. This indicates that, under guaranteed convergence condition, the larger ω_1 is selected (> 2), the faster the convergence rate of computation and the higher the accuracy of data.
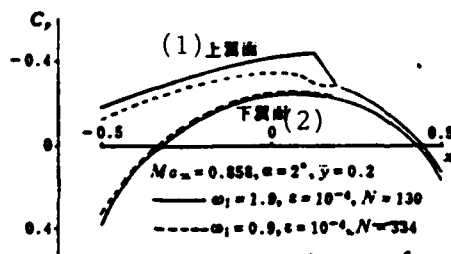

Fig. 4. Influence of relaxation parameters on computed results
Key: (1) upper wing surface; (2) lower wing surface.

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Abstract

A three-dimensional computation of a jet in a crossflow has been performed on a microcomputer. The SIMPLE method is adopted with some modifications. A special computer code is developed in conformity with the limited memories of the microcomputer. Velocity and presure distributions are given and compared with the available experimental data satisfactorily.

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NUMERICAL COMPUTATION OF THE THREE-DIMENSIONAL VISCOUS INTERFERENCE
FLOW FIELD OF A VERTICAL JET IN A CROSSFLOW

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A vertical jet shot out of a flat plate will interfere with a
low speed incident flow parallel to the flat plate (crossflow), caus-
ing complex flow phenomena to occur. The jet is deflected by the
crossflow, and its velocity is rapidly reduced; the incident flow
is blocked and drawn by the jet (Fig. 1). The jet and the incident
flow interreact with each other causing two symmetrical vortex cores
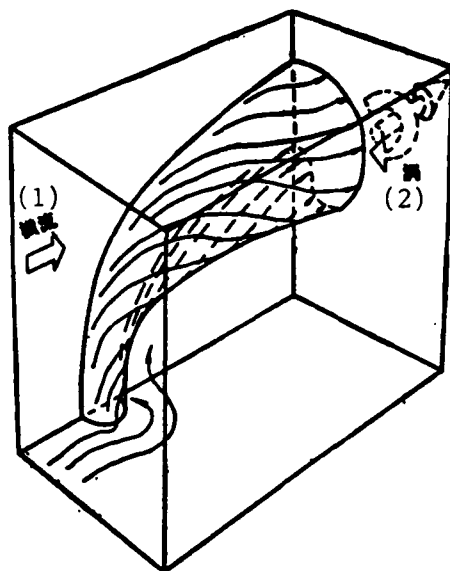to develop under the jet.

Fig. 1. Flow field of a vertical jet interferring with a crossflow
and its computation domain
Key: (1) crossflow; (2) vortex.

The vortex caused by this interference develops along the direction

of the wall surface and it can be used as a means to control the

attach surface layer[1]. It can also be applied to heat transfer

engineering and other areas. There have been numerous experimental

studies on this kind of flow both in our country and abroad[1-4].

Due to the difficulties in measurement, it is generally difficult

to obtain detailed data of the entire flow field, especially near

the nozzle. It is impossible to conduct computation using two-dimen-

sional or nonviscous method because of the complexity of the flow;

the full three-dimensional viscous numerical computation method must

be used. This kind of flow possesses strong three-dimensional mixing

effects and needs to use appropriate turbulence model. According

to the principles in References [6, 7], we developed a computer program

and performed computation.

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I. Basic Equations

The basic equation of a three-dimensional, steady, incompressible, time-averaged Reynold number turbulent flow in the rectangular coordinates is: continuity equation

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

The momentum equation (with the assumption of vortex viscosity)

$$\rho u_j \frac{\partial u_j}{\partial x_i} = -\frac{\partial p^*}{\partial x_j} + \frac{\partial}{\partial x_i}\left[\mu_e\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{2}$$

where the effective viscosity μ_e is defined as

$$\mu_e = \mu_l + \mu_t \tag{3}$$

where μ_l is the laminar viscosity. μ_t is the turbulent viscosity and is determined by the following turbulence model

$$\mu_t = C_\mu \rho K^2/\epsilon \tag{4}$$

where K is the turbulence kinetic energy. ϵ is the dissipation rate of K and is determined by the following equations

$$\rho u_i \frac{\partial K}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\mu_t}{\sigma_t} \frac{\partial K}{\partial x_i}\right) + G - \rho\epsilon \tag{5}$$

$$\rho u_i \frac{\partial \epsilon}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\mu_t}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_i}\right) + (C_1 G - C_2 \rho\epsilon)\frac{\epsilon}{K} \tag{6}$$

where G is the generation rate of the turbulence kinetic energy

$$G = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} \tag{7}$$

The coefficients used in the equations are adopted from the data recommended by Reference [8]:

$$C_\mu = 0.09, \ C_1 = 1.44, \ C_2 = 1.92, \ \sigma_k = 1, \ \sigma_\epsilon = 1.3 \tag{8}$$

Since the above turbulence model is used, p* in Equation (2) is

$$p^* = p + \frac{2}{3}\rho K$$

where 2/3 K can be regarded as turbulent pressure term.

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II. Solution Method

The method used by this paper is based primarily upon the SIMPLE method[7]. Said method adopts the iteration solution method. The three velocity components u, v and w used to solve the x-direction momentum equation are all old values; u used to solve the y-direction momentum equation is a new value; and u and v used to solve the z-direction momentum equation are new values. It was found during actual computation that this kind of nonconforming value character was an important factor which caused divergence in solving the equation. Therefore, when we were developing the program the newly obtained u and v values were temporarily stored and old u, v and w values were used to perform iteration for all three momentum equations, thereby guaranteeing the conforming value character.

In order to increase efficiency and lower cost, the existing INTEL-86-330 microcomputer was adopted to perform computation. Said program requires more memory. The example in this paper requires more than 100 giga bytes in memory at least, and the available memory space of the said microcomputer is only a little more than 100 K. Therefore, during the development of the program the technique of exchanging with secondary storage was adopted. Every three-dimensional group was divided into a dozen or so two-dimensional group and stored in hard disk. During computation they were called in separately by a subroutine to take part in the iteration and then restored. This greatly saved memory space. Yet as a price, the computation speed dropped about 20 fold.

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III. Example

The experimental conditions in Reference [3] for the incident
flow and geometric data are used. The incident flow velocity is
39 m/sec and jet velocity is 156 m/sec with a velocity ratio of 4.
The symmetry of flow field only considers half of the flow field
cut by the symmetric plane, as shown in Fig. 1. The computation
domain is a box-shaped region with its bottom connected to the flat
plate. One of the sides is the symmetric plane and the other four
sides are the upstream, downstream boundaries and infinite boudnaries,
respectively. The grid is in the form of rectangular coordinates:
x-direction (flow direction) has 20 rows of grid, z-direction (per-
pendicular to the plate) has 16 rows and y-direction has 12 rows
for a total of 3840 grid points (measurement points). In order to
properly simulate the details of flow field near the jet nozzle,
a variable spacing grid is used to give dense grid near the jet nozzle
and sparse grid for further flow field. Using the nozzle diameter
D as the reference, the upstream boundary is 3.5 D from the center
of the nozzle and the downstream boundary is 8 D from the center
of the nozzle. The z-direction infinite boundary is about 12 D from
the plate and the side infinite boundary is about 5 D from the symmet-
ric plane. The method in Reference [6] is adopted. Six grid points
are used to simulate a semi-circle jet nozzle. Meanwhile the total
area corresponding to the six grid points is made to be in agreement
with the area of the semi-circle jet nozzle.

The upstream boundary defines the incident flow velocity. The normal velocities on the infinite boundary, the symmetric plane boundary and the flat plate are all assumed to be zero. The jet velocity is defined by the jet nozzle. The gradient of the tangential velocities v and w on the downstream boundary are assumed to be zero, yet u is adjusted according to the total flow rate on the basis of its value on the previous row. The wall surface is processed by adopting the wall surface function method[8].

The standard for computation convergence is set at when both the corresponding remainders of the momentum and continuity equations reach 5%. This example required 90 steps and spent 170 hours on the microcomputer.

V. Results and Their Discussions

In order to verify the computation, the results are compared with the test data of Reference [3, 4, 5]. Figure 2 shows the velocity contours on the symmetric plane. In the figure, the line with indexing points is the jet center line drawn by using the peaks of the contours, and it can be seen that the line is in good agreement with the test data. See Fig. 3 for the decay of jet velocity. In the figure, W_{max} is the jet velocity at the jet nozzle, s is the natural coordinates along the jet, W_j is the jet velocity along s. The computed velocity decays faster than the test value, but the difference is not distinctive and both show that the rate of velocity decay follows a rule of first small, then large, and again from large back to small. Figure 4 is the projection of vortex core contour on the symmetric

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plane. The vortex core trace derived from the computed values is more complicated. The low pressure centers basically coincide with the vortex core contour, otherwise the centripetal force that makes the air streamlines turn vortically will not be generated. The curve in Fig. 4 is the projection of the low pressure centers on the symmetric plane, i.e., the approximation of the vortex core contour, and matches well with the vortex core test values. Figure 5 is the comparison of computed values (left half) and test values (right half) of the pressure coefficient distributions on the flat plate. Judging from the trend of pressure distributions, the computed values of pressure rise behind the jet nozzle on the flat plate are faster than the test values.
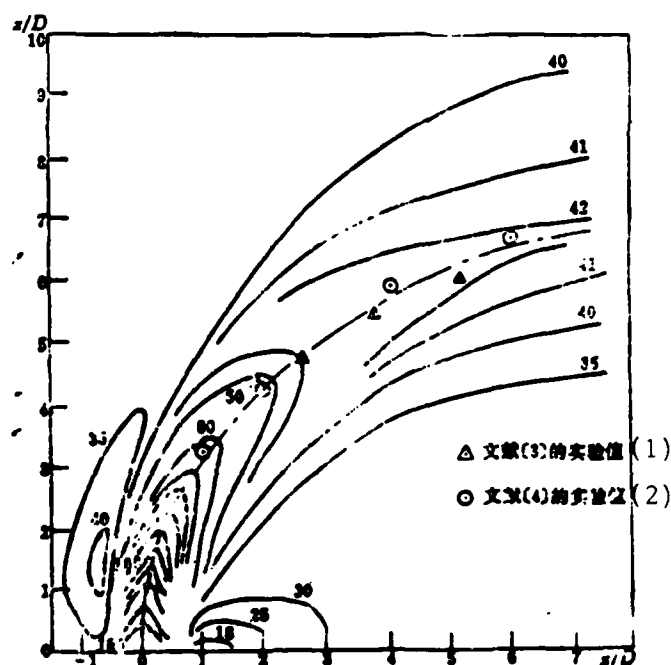

Fig. 2. Velocity distributions and the location of jet center line on the symmetric plane
Key: (1) test values of Reference [3]; (2) test values of Reference [4].
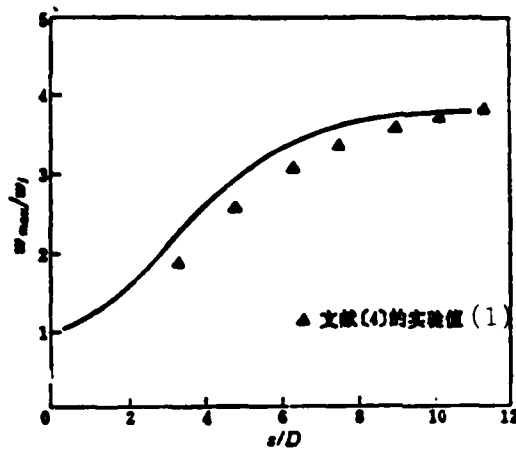
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Fig. 3. Rule of velocity decay on the jet center line
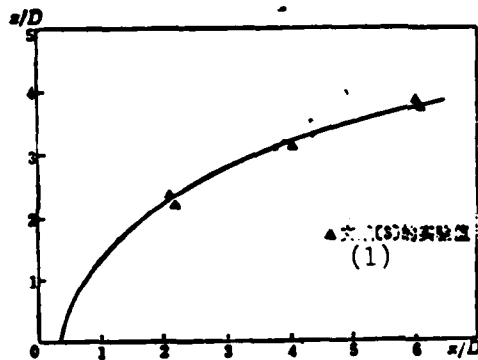Key: (1) test values of Reference [4].

Fig. 4. Projection of the location of vortex core on the symmetric plane
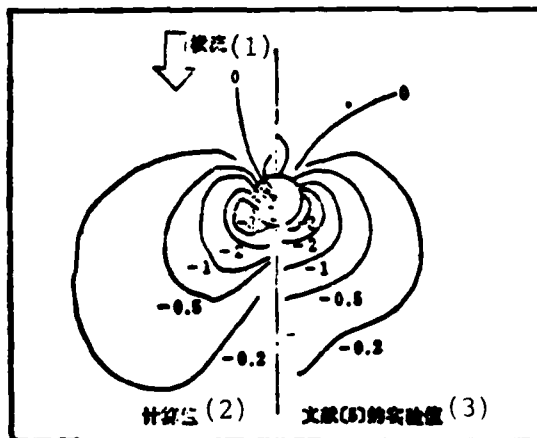Key: (1) test values of Reference [3].

Fig. 5. Pressure coefficient distribution on the flat plate
Key: (1) Crossflow; (2) Computed values; (3) test values of Reference [5].

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The three-dimensional viscous flow computation method introduced by this paper can compute detailed data of the interference flow field of a vertical jet in an incident flow. The comparison with test results shows that the computed results are more accurate, especially the locations of jet contour, vortex core contour, etc. The fact that the computed jet decay is a little faster and the computed pressure rise behind the jet nozzle is faster indicates that the numerical dissipation might be larger, but not enough to cause significant influence on the prediction of the entire flow field. Therefore, the said numerical method can be an effective tool for analyzing flow field.

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